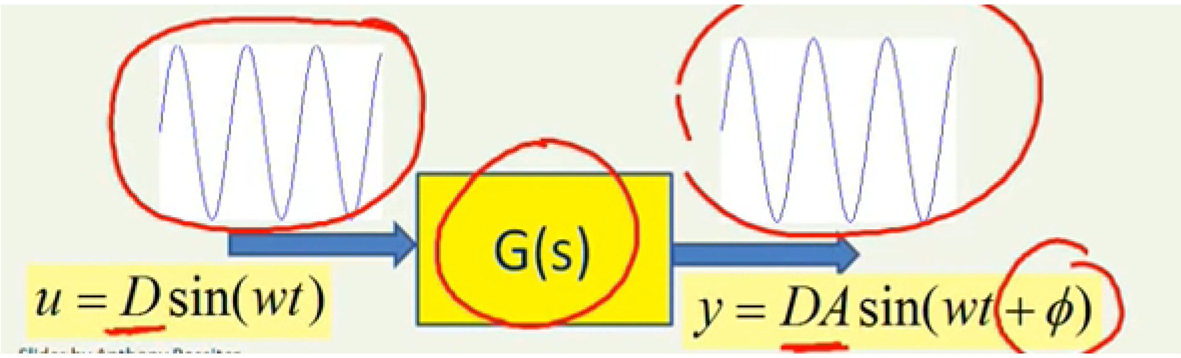
Nyquist Stability

Background



Bode diagrams separated the gain and phase of the complex number $G(j\omega)$

Bode plot review

$G(s) = a + bj$

$|G(s)| = \sqrt{a^2 + b^2}$

z^{-1} = a + bj

$[r[cos(\theta) + isin(\theta)]^{-1} = [R[cos(\alpha) + isin(\alpha)]$

$[r[cos(\theta) + isin(\theta)] = [R^{-1}[cos(\alpha) + isin(\alpha)]^{-1}$

$R = \frac{1}{r}$

$\frac{1}{cos(\alpha)+isin(\alpha)}$

$cos(-\alpha) + isin(-\alpha)$

Phase $= -\alpha = -arctan(\frac{b}{a})$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}[cos(\theta_1 - \theta_2) + isin(\theta_1 - \theta_2)]$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\frac{cos(\theta_1)+isin\theta_1}{cos(\theta_2)+isin\theta_2}$

$|\frac{z_1}{z_2}| = \frac{r_1}{r_2}$

$\theta_1 - \theta_2 = \arctan(\frac{b_1}{a_1}) - \arctan(\frac{b_2}{a_2})$

Multiplication and division of complex numbers

Students are reminded of the following rules for complex numbers.

1. Modulus of the product is the product of the moduli.
2. Phase (or argument) of the product is the sum of the phases.

$$|wzy| = |w||z||y|; \quad \left|\frac{wz}{ykm}\right| = \frac{|w||z|}{|y||k||m|}; \quad \left|\frac{w^2 z}{y^3 k}\right| = \frac{|w|^2|z|}{|y|^3|k|}$$

$$\angle wzy = \angle w + \angle z + \angle y; \quad \angle \frac{w^2 z}{y^3 k} = 2\angle w + \angle z - 3\angle y - \angle k$$

Example 1

$G = \frac{4}{s^2+3s+2}$

$G(j\omega) = \frac{4}{3j\omega+2-\omega^2}$

$|G(j\omega)| = \frac{4}{(\sqrt{(2-\omega^2)^2+9\omega^2}}$

Phase $= -tan^{-1}(\frac{3\omega}{2-\omega^2})$

Example 2

$G = \frac{2s+1}{s^3+s^2+3s+2}$

Find the frequency response (gain and phase definitions) for G(s). [$s = jw$]

$$G(jw) = \frac{2jw+1}{(jw)^3 + (jw)^2 + 3jw + 2} = \frac{2jw+1}{j(3w - w^3) + 2 - w^2}$$

$$|G(jw)| = \frac{\sqrt{4w^2+1}}{\sqrt{(3w-w^3)^2 + (2-w^2)^2}}$$ [NOT NICE]

$$\angle G(jw) = \tan^{-1}\frac{2w}{1} - \tan^{-1}\frac{(3w-w^3)}{2-w^2}$$ [YUCK]

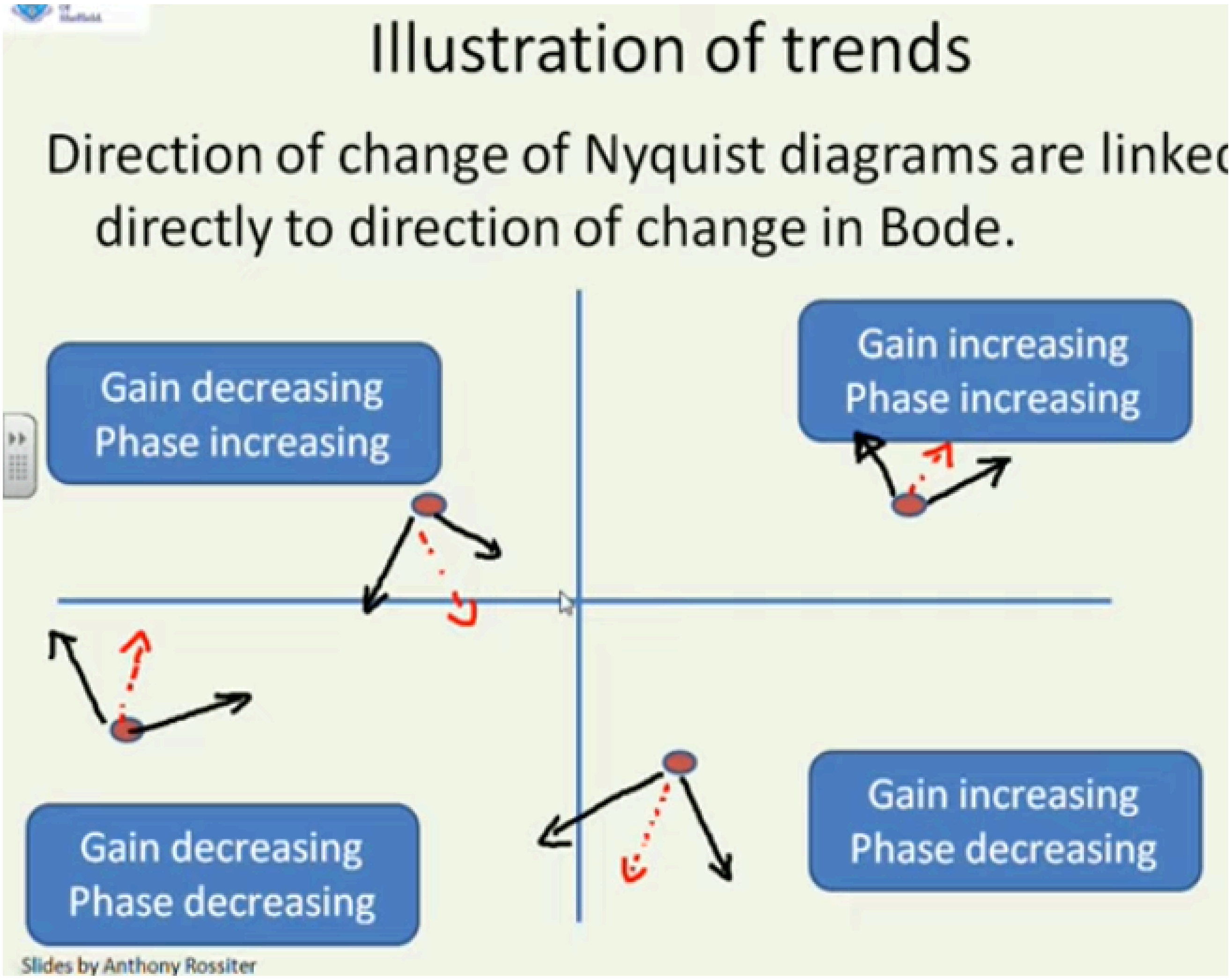
Nyquist plot

Bode diagrams separate the gain and phase of the complex number G(jw)

Nyquist diagrams is a plot of G(jw) on the complex plane.

Guidelines

1. Phase is reducing (or becoming more negative) – plot is moving clockwise.
2. Phase is increasing – plot is moving anti-clockwise.
3. Gain is reducing – plot is moving towards origin.
4. Gain is increasing – plot is moving away from the origin.



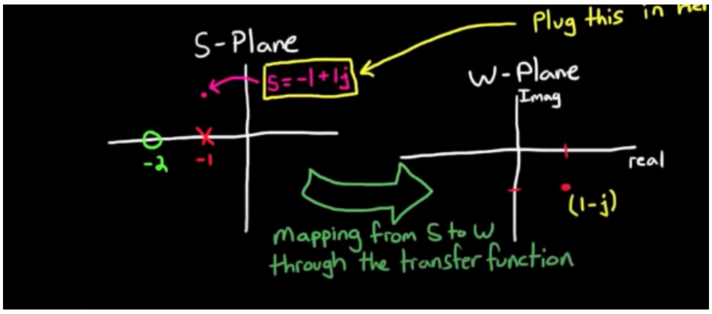
Example 1

Cauchy argument's principal

a) Transfer function $= \frac{s+2}{s+1}$

s = -1 +j

New complex number $1 - j$

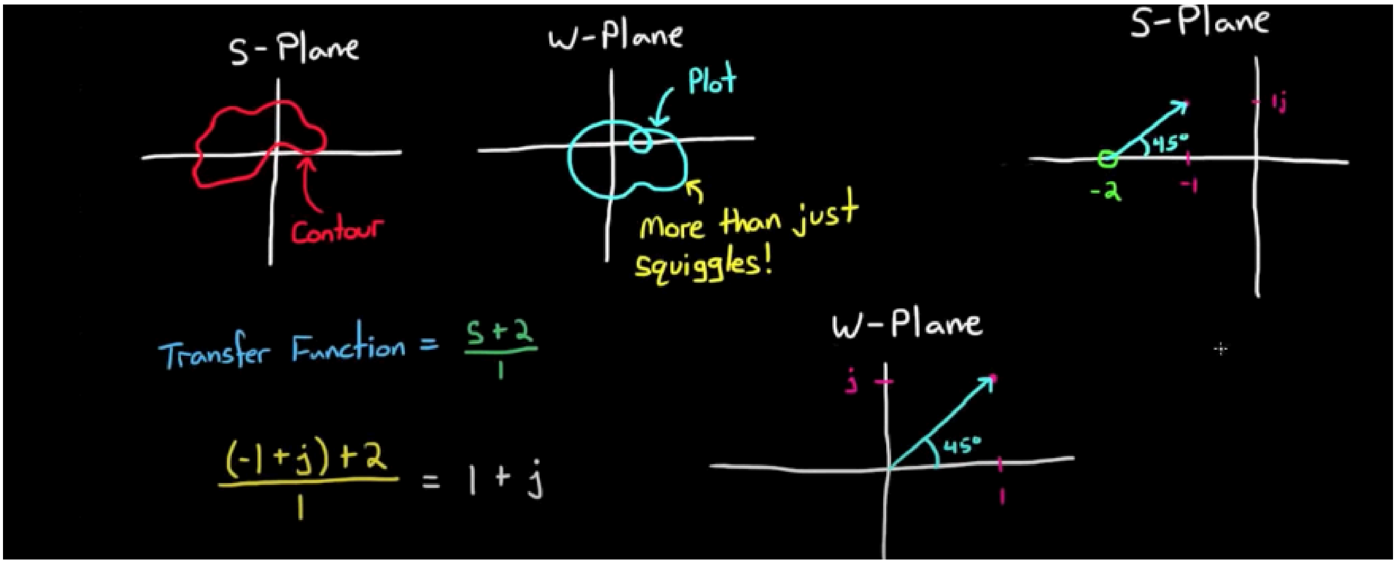


b)

Transfer function $= \frac{s+2}{1}$

$s = -1 + j$

Transfer function $= 1 + j$



same phasor.

Example 2

$G(s) = \frac{1}{1+s}$

$G(s) = \frac{1}{1+j\omega} = \frac{1-j\omega}{1+\omega^2}$

Example 3

$G(s) = \frac{6}{(s+1)(s+3)}$

$G(jw) = \frac{6}{(j\omega+1)(j\omega+3)}$

$G(jw) = \frac{6(j\omega-3)}{(j\omega+1)(-\omega^2-9)}$

$G(jw) = \frac{6(3-j\omega)(1-j\omega)}{(((1+\omega^2)(\omega^2+9)}$

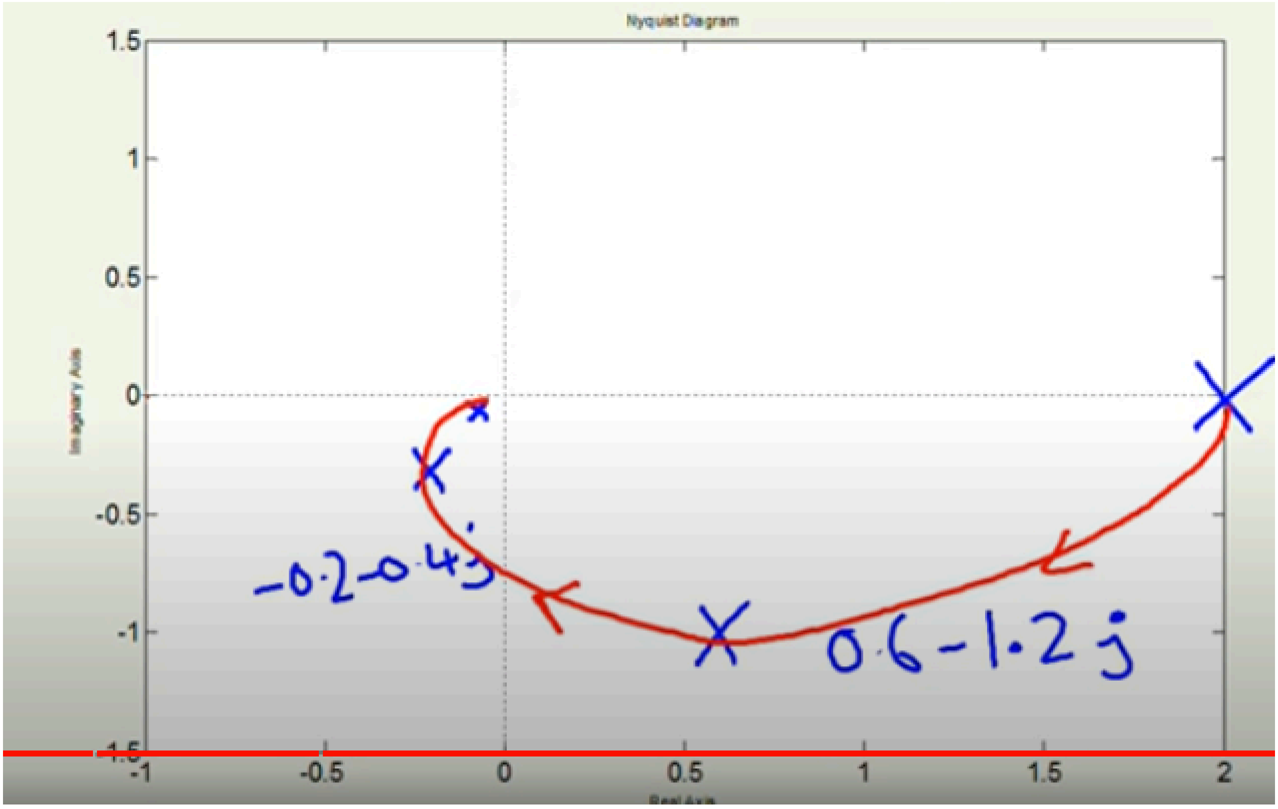
$G(jw) = \frac{6([3+\omega^2]-j[4\omega])}{(((1+\omega^2)(\omega^2+9)}$

$G(jw) = \frac{6\sqrt{((1+\omega^2)(\omega^2+9)}}{(((1+\omega^2)(\omega^2+9)}$

4th Quadrant

Phase = $-arctan(\frac{\omega}{1}) - arctan(\frac{\omega}{3})$

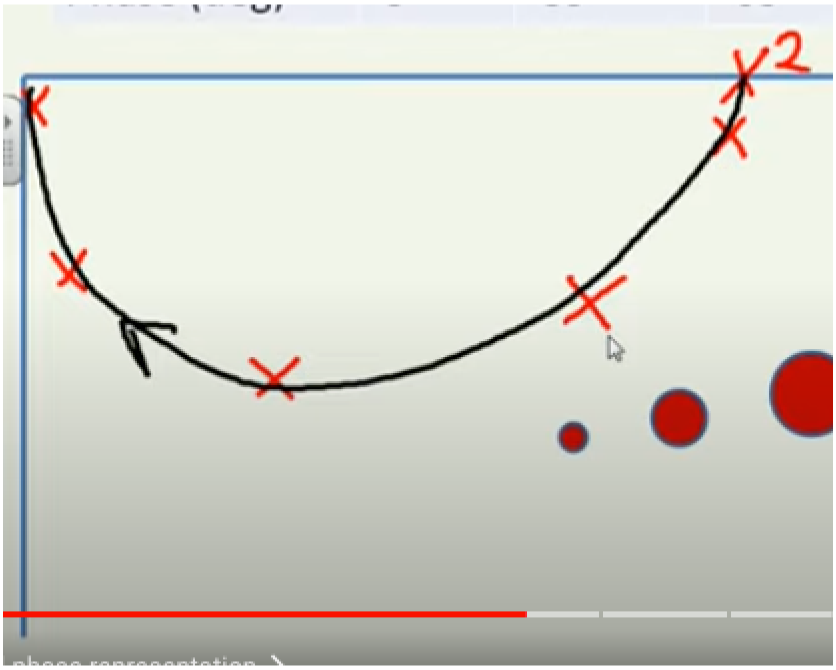
	GAIN	Phase	
w=0	2	0	2
w=1	1.34	-63	0.6 - 1.2j
w=3	0.45	-116	-0.2 -0.4j
w=10	0.06	-157	-0.05 - 0.02j



Example 4

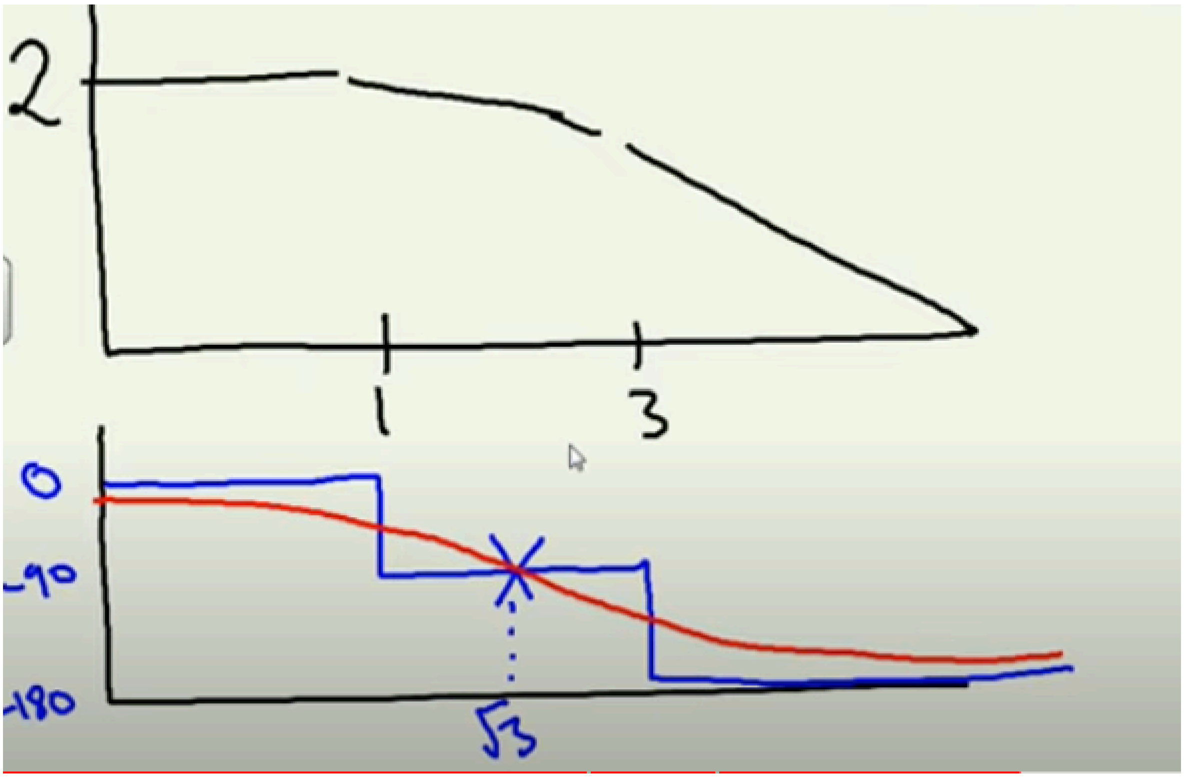
Sketch the Nyquist for G = (s+2)/[(s+1)(s+1)]

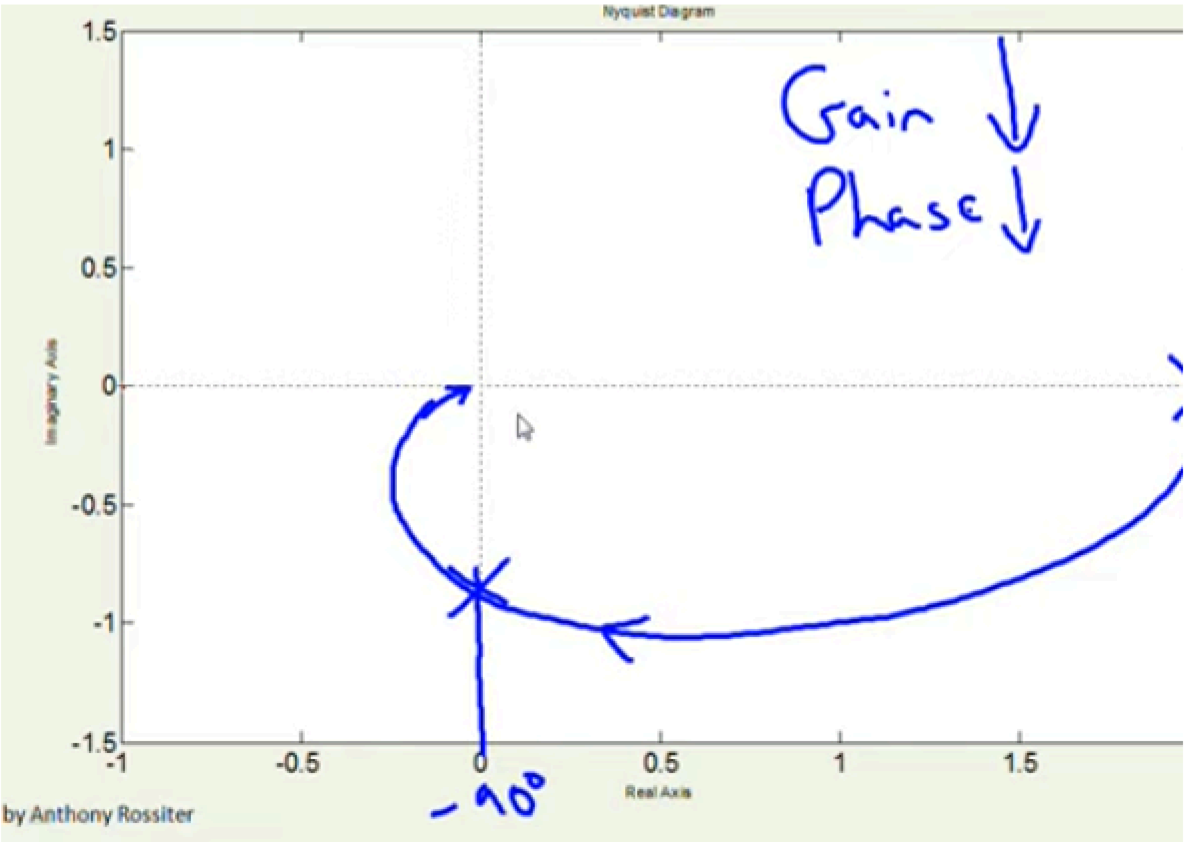
Freq (rad/s)	0.1	0.5	1	2	10
Gain	1.98	1.65	1.12	0.57	0.1
Phase (deg)	-9	-39	-63	-82	-90



Example 5

Sketch a bode plot of $G(s) = \frac{6}{(s+1)(s+3)}$





Example 6

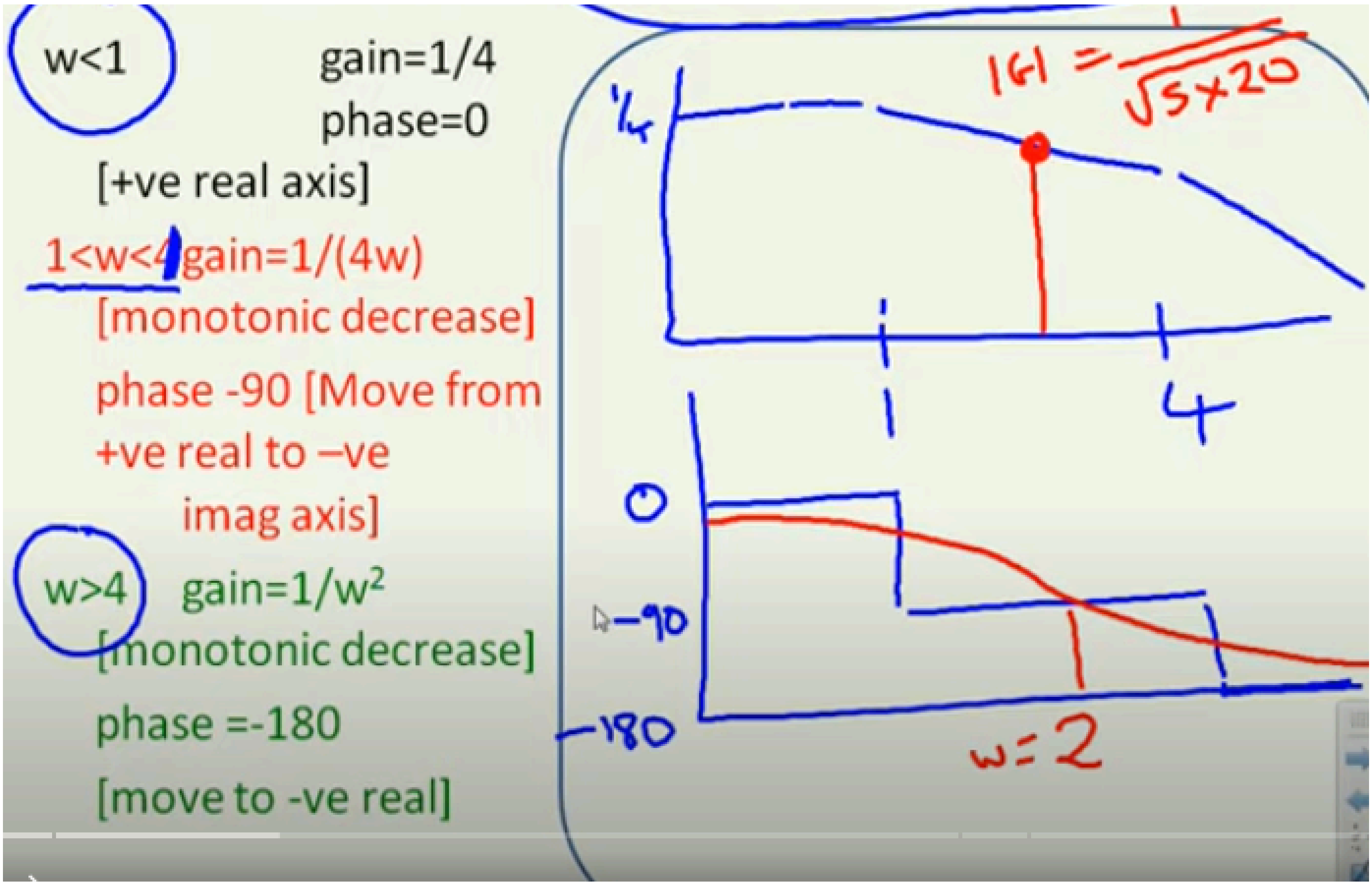
$$G = \frac{1}{(s+1)(s+4)}$$

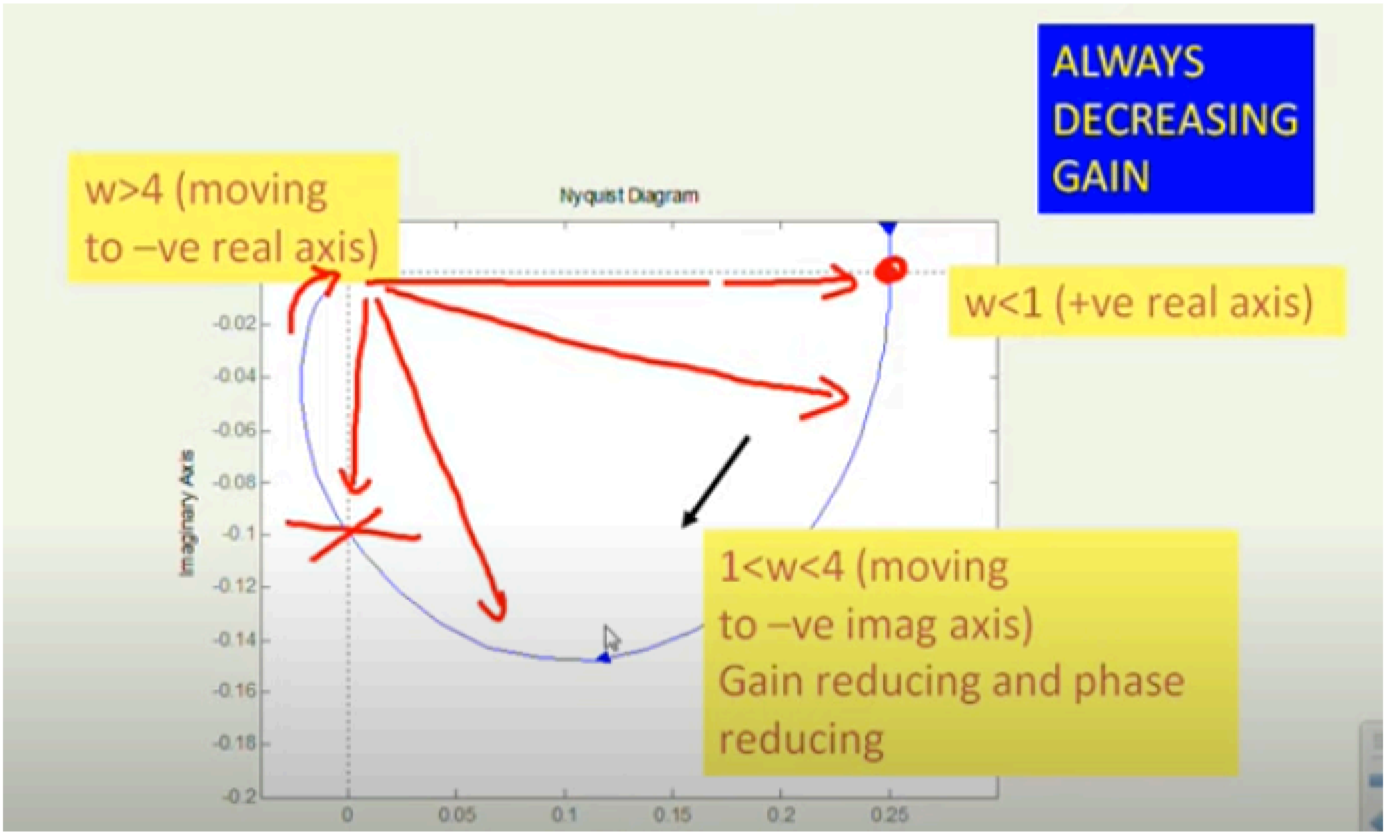
$$G = \frac{-0.3333}{(s+1)} + \frac{0.333}{(s+4)}$$

$$G = \frac{-0.3333}{(\omega j+1)} + \frac{0.333}{(\omega j+4)}$$

$$G = \frac{-0.3333(\omega j-1))}{(-\omega^2-1)} + \frac{0.333(\omega j-4)}{(-\omega^2-16)}$$

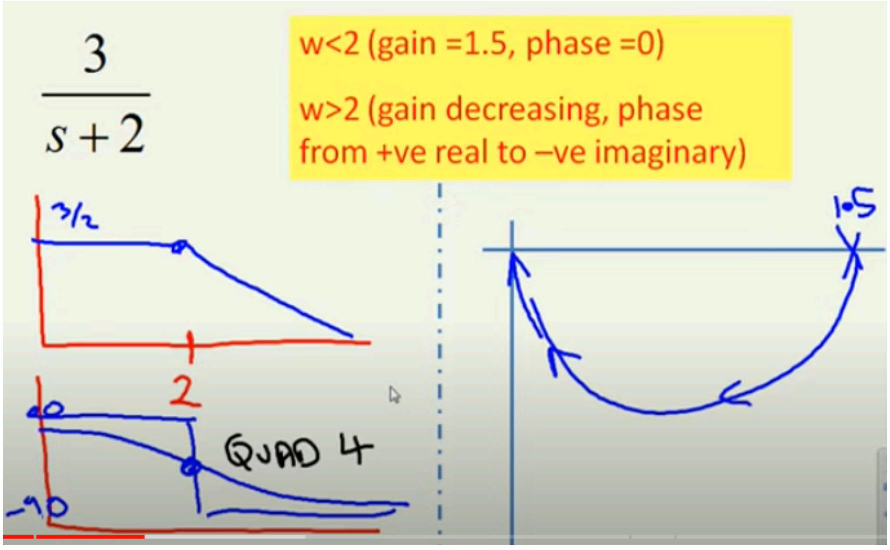
$$G = -0.3333\frac{(1-\omega j))}{(\omega^2+1)} + \frac{0.333(4-\omega j)}{(\omega^2+16)}$$





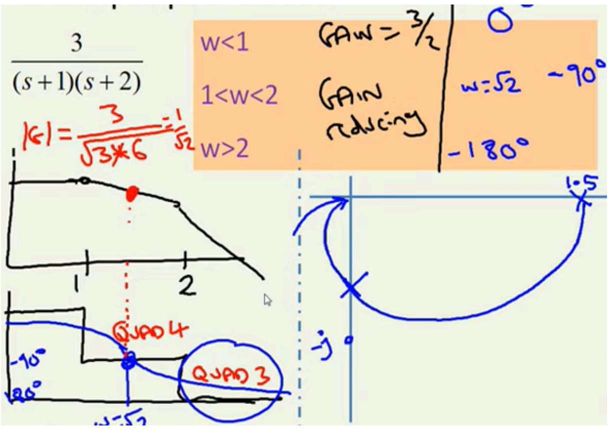
Example 7

$$\frac{3}{s+2}$$



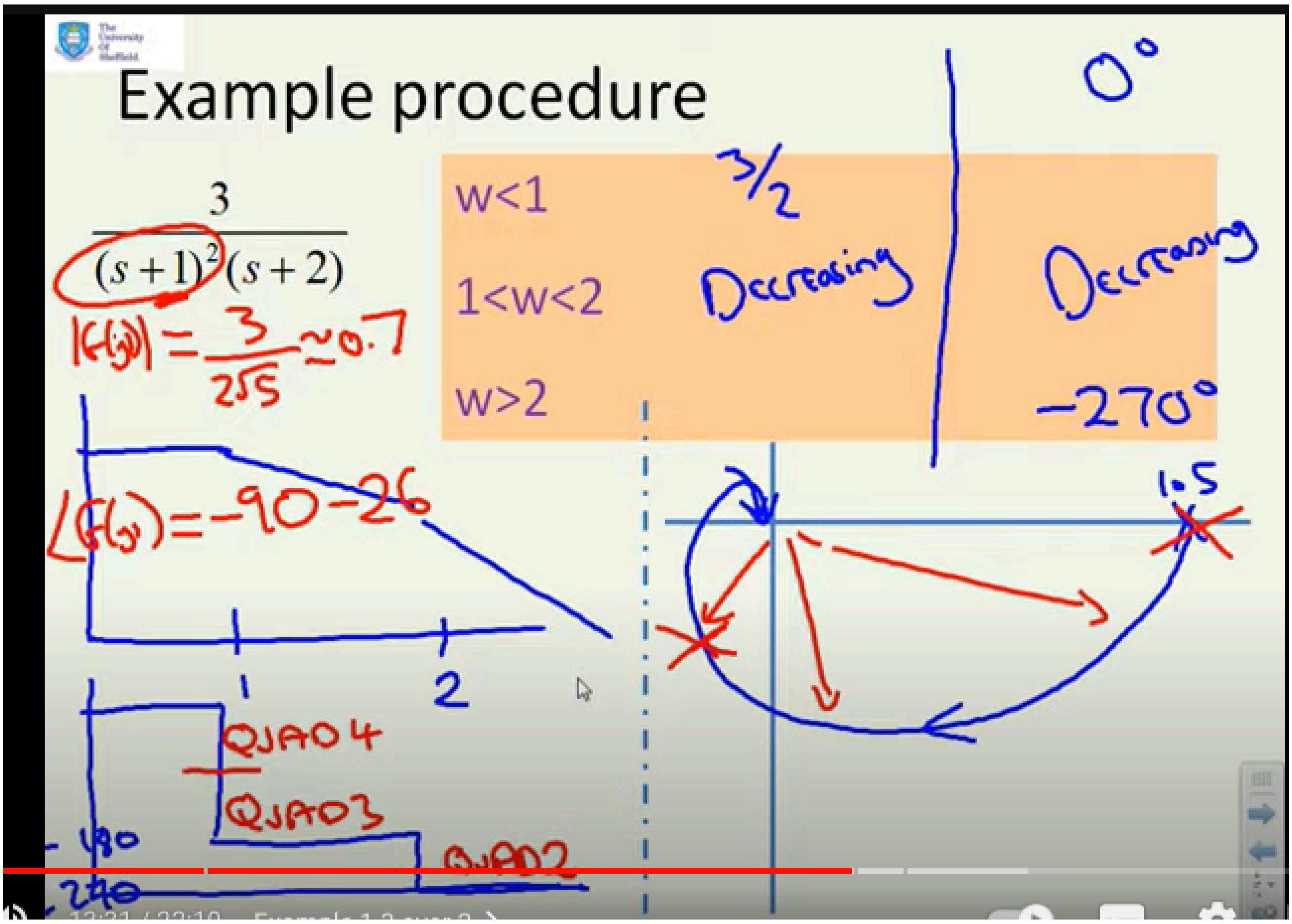
Example 8

$$\frac{3}{(s+1)(s+2)}$$



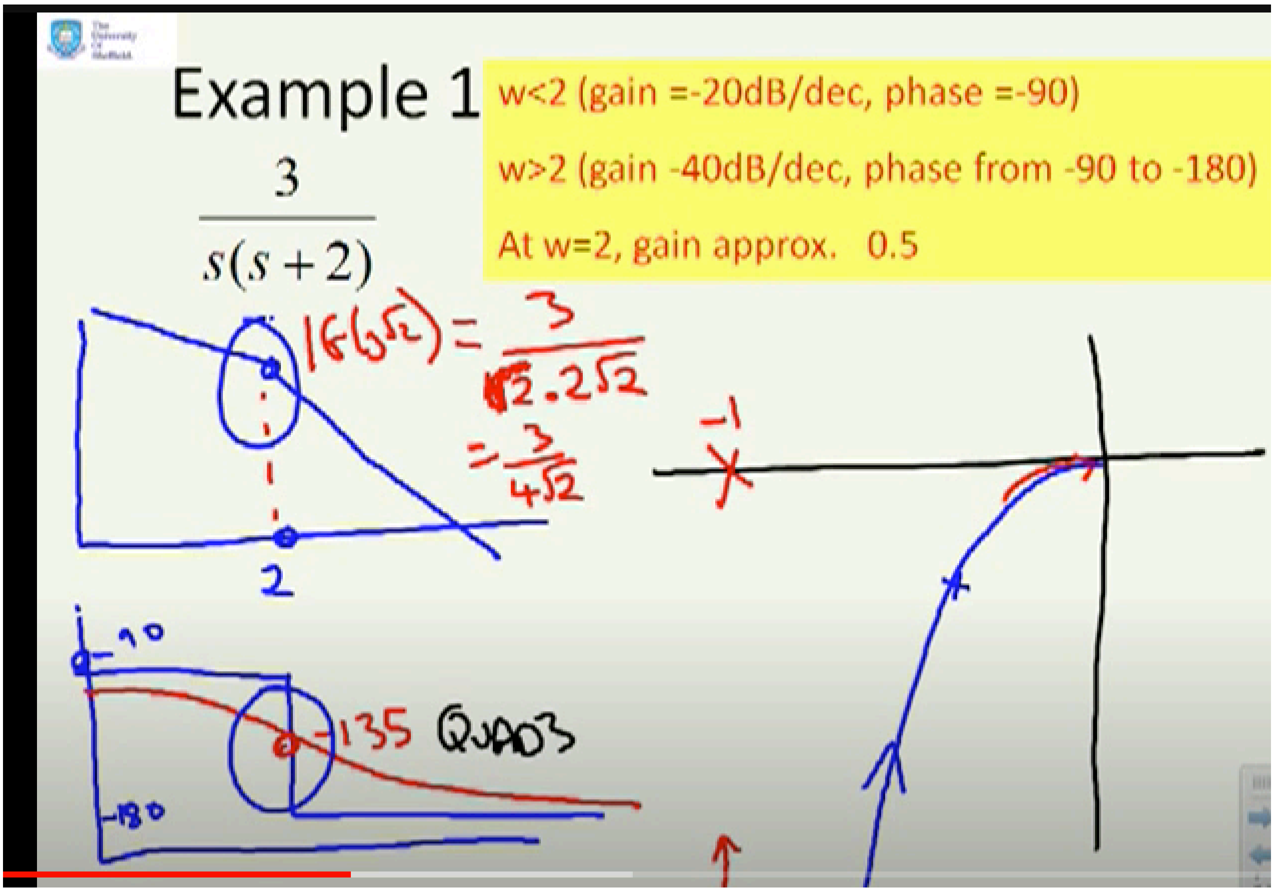
Example 9

$$\frac{3}{(s+1)^2(s+2)}$$



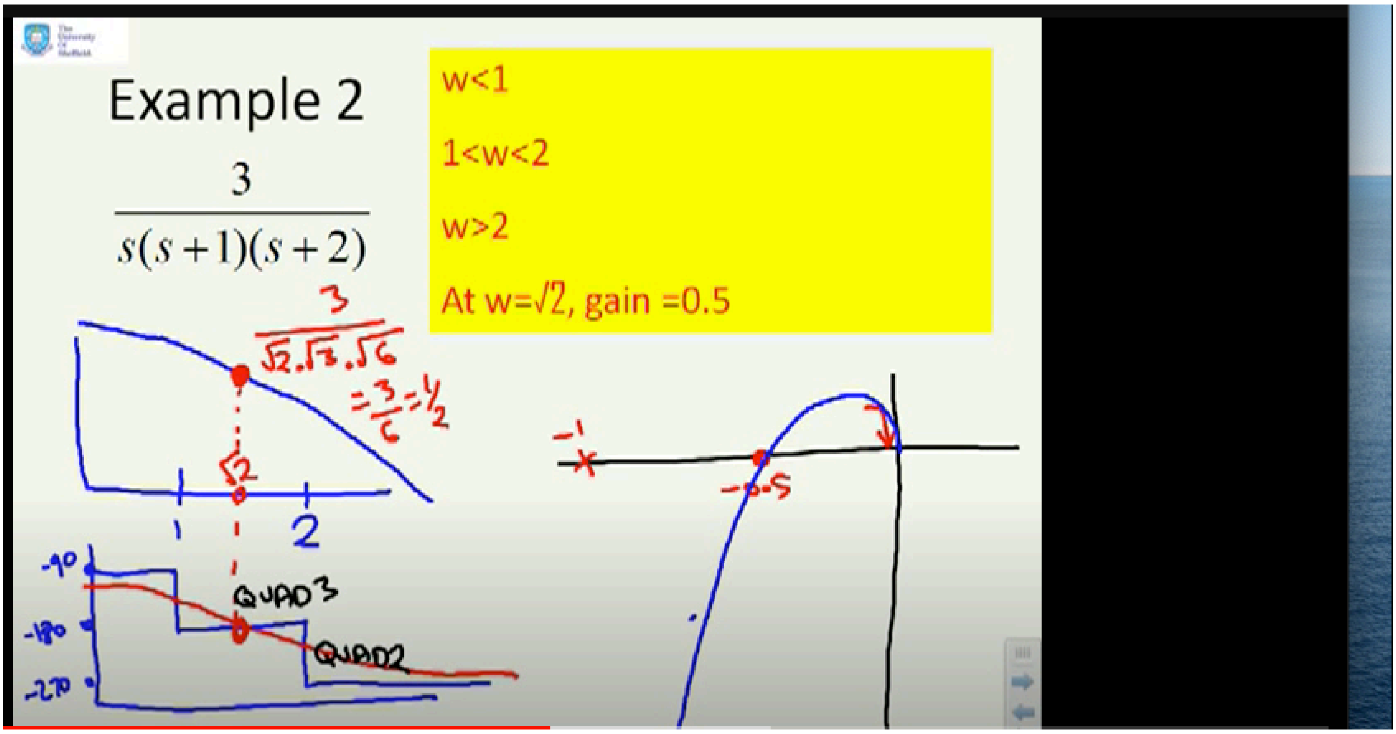
Example 10

$$\frac{3}{s(s+2)}$$



Example 11

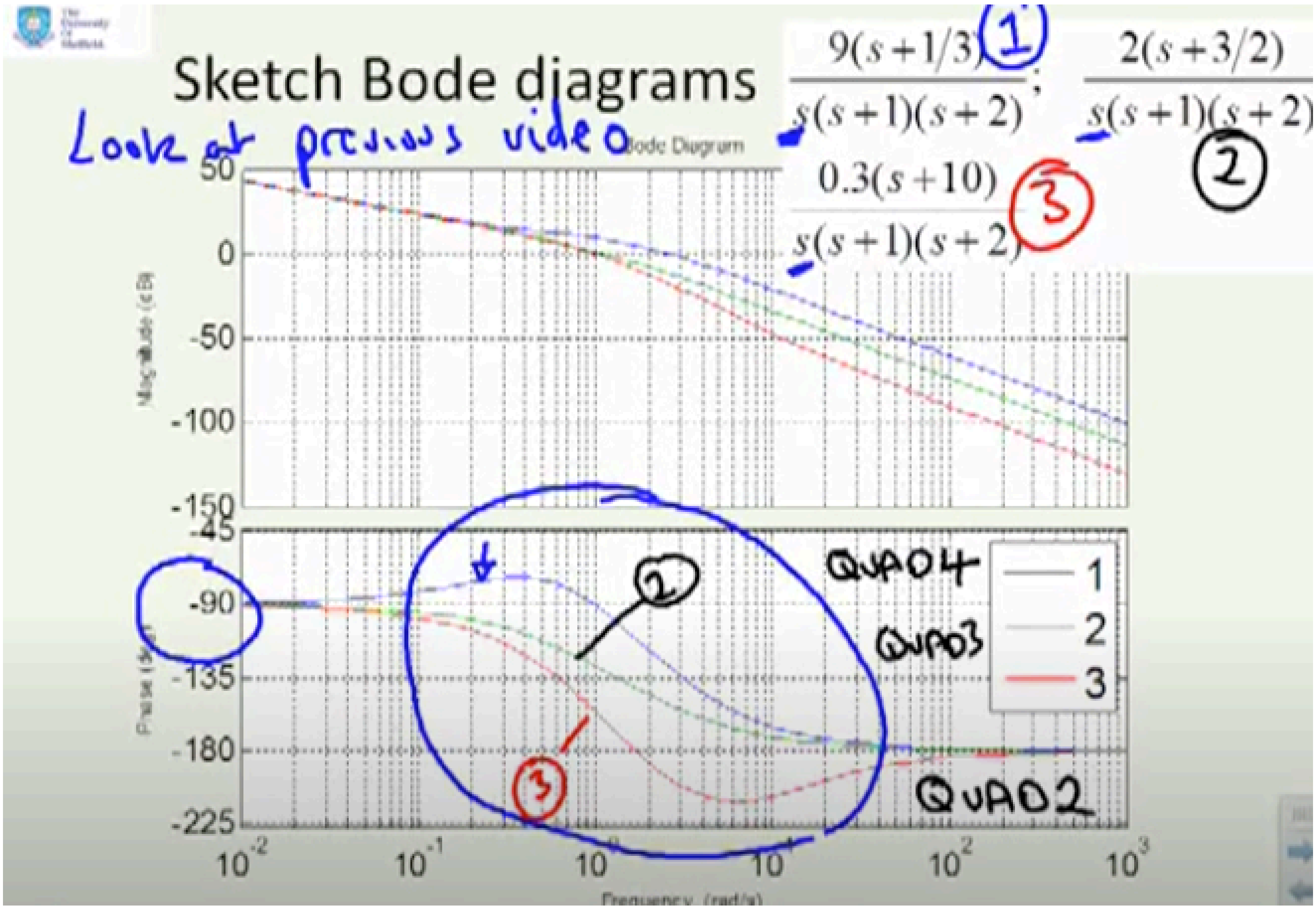
$$\frac{3}{s(s+1)(s+2)}$$

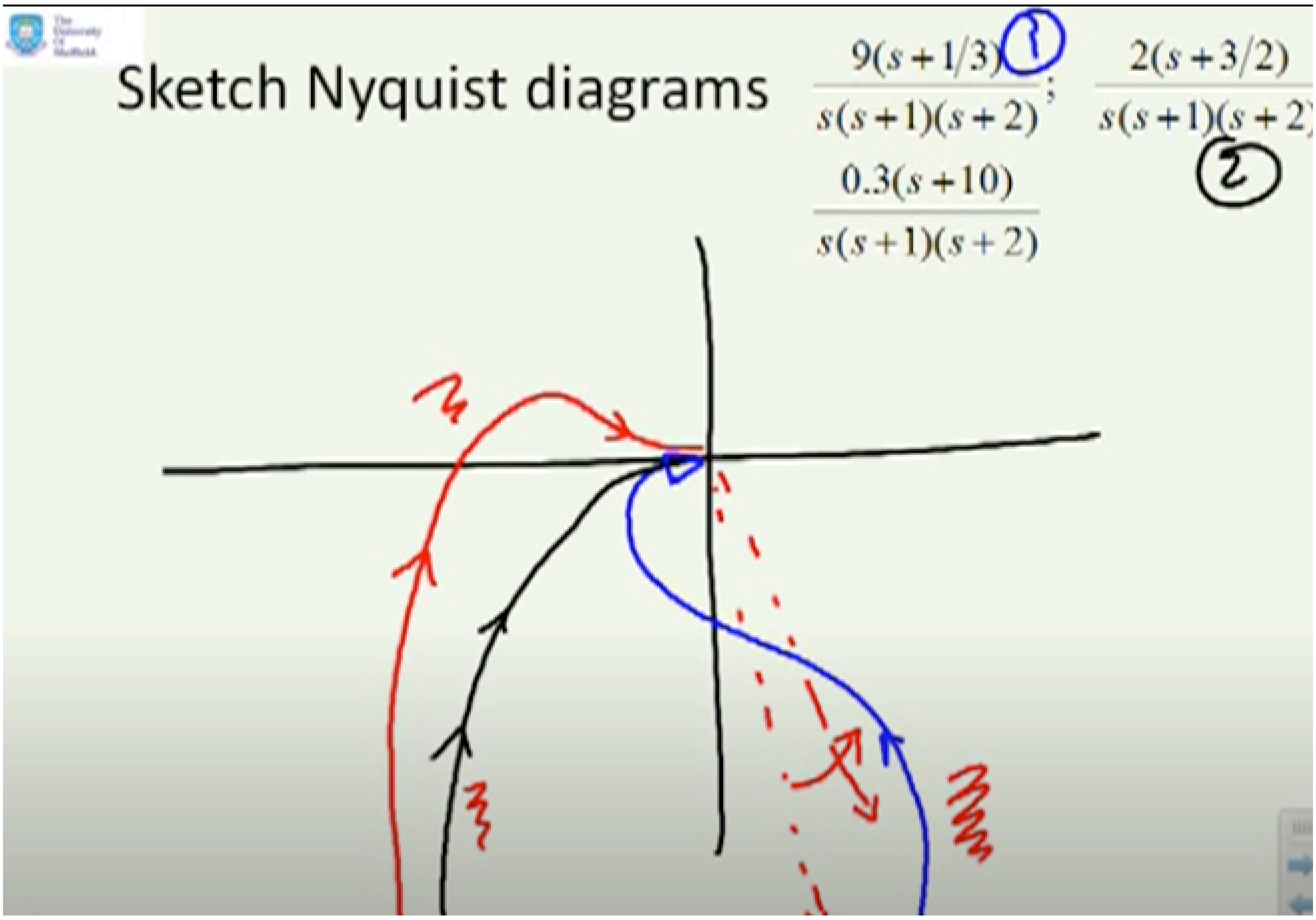


Screenshot 2024-04-11 at 18.05.45.png

Screenshot 2024-04-11 at 19.31.17.png

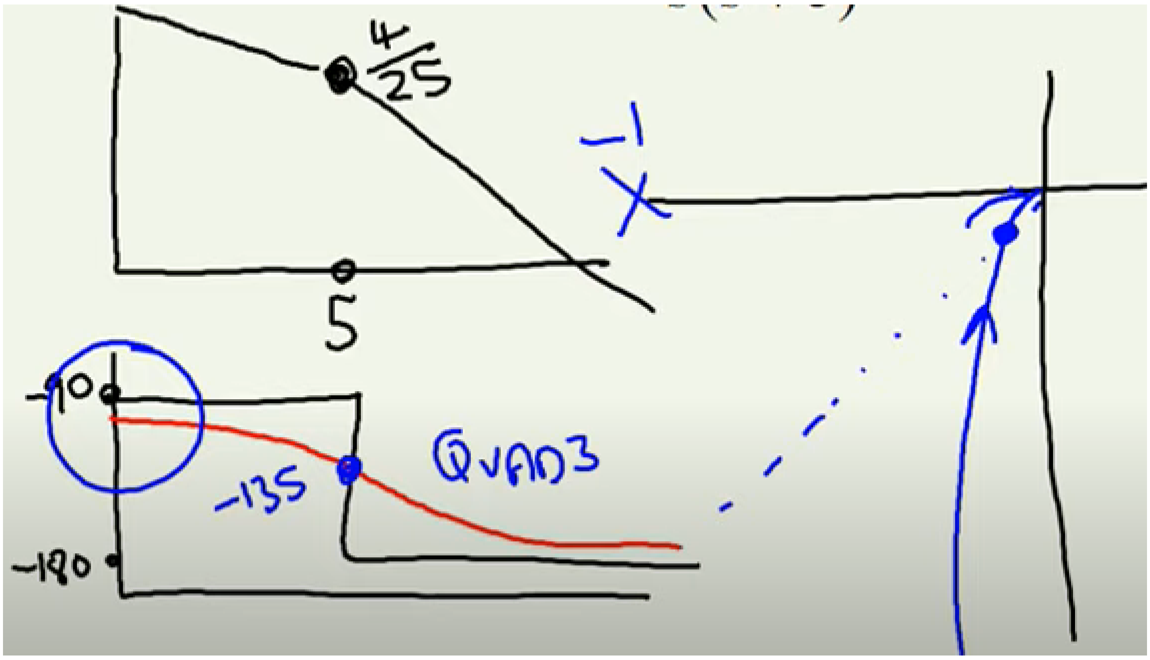
Example 12





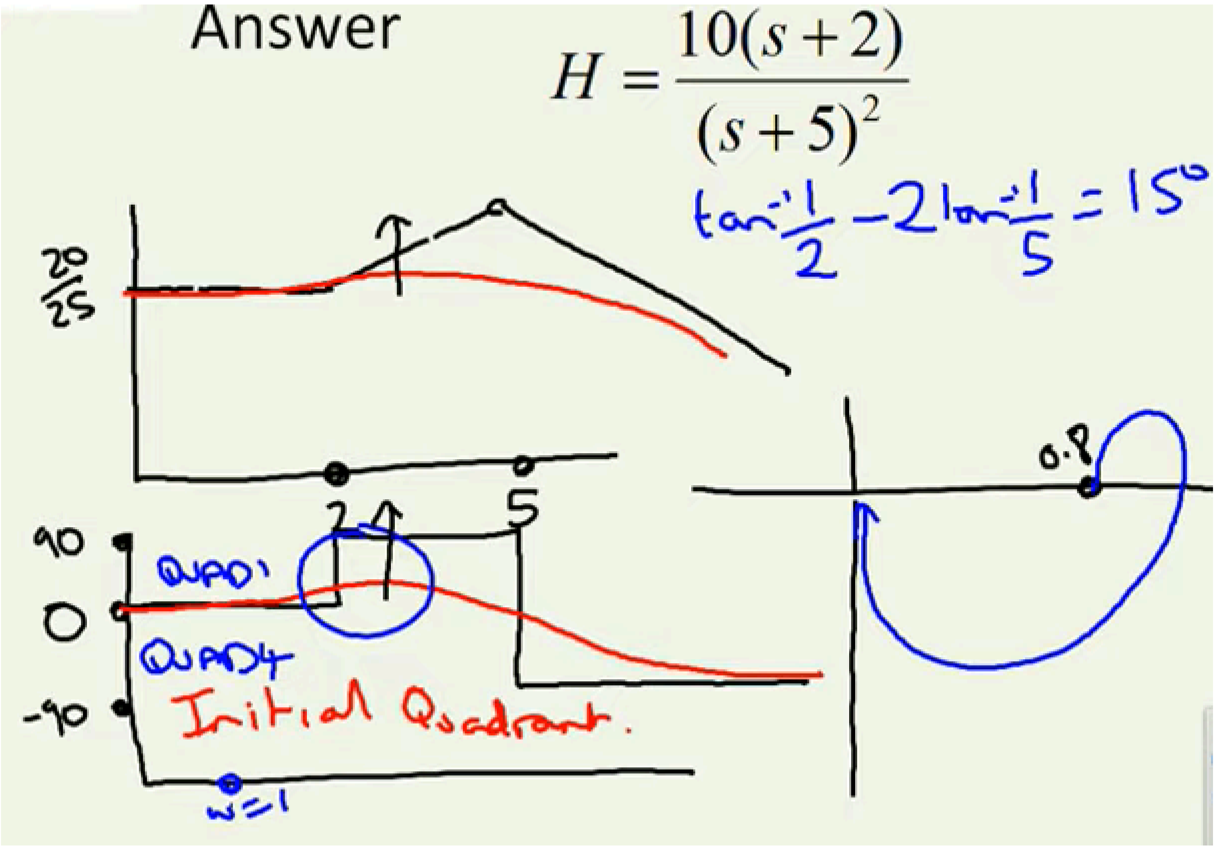
Example 13

$$\frac{4}{s(s+4)}$$



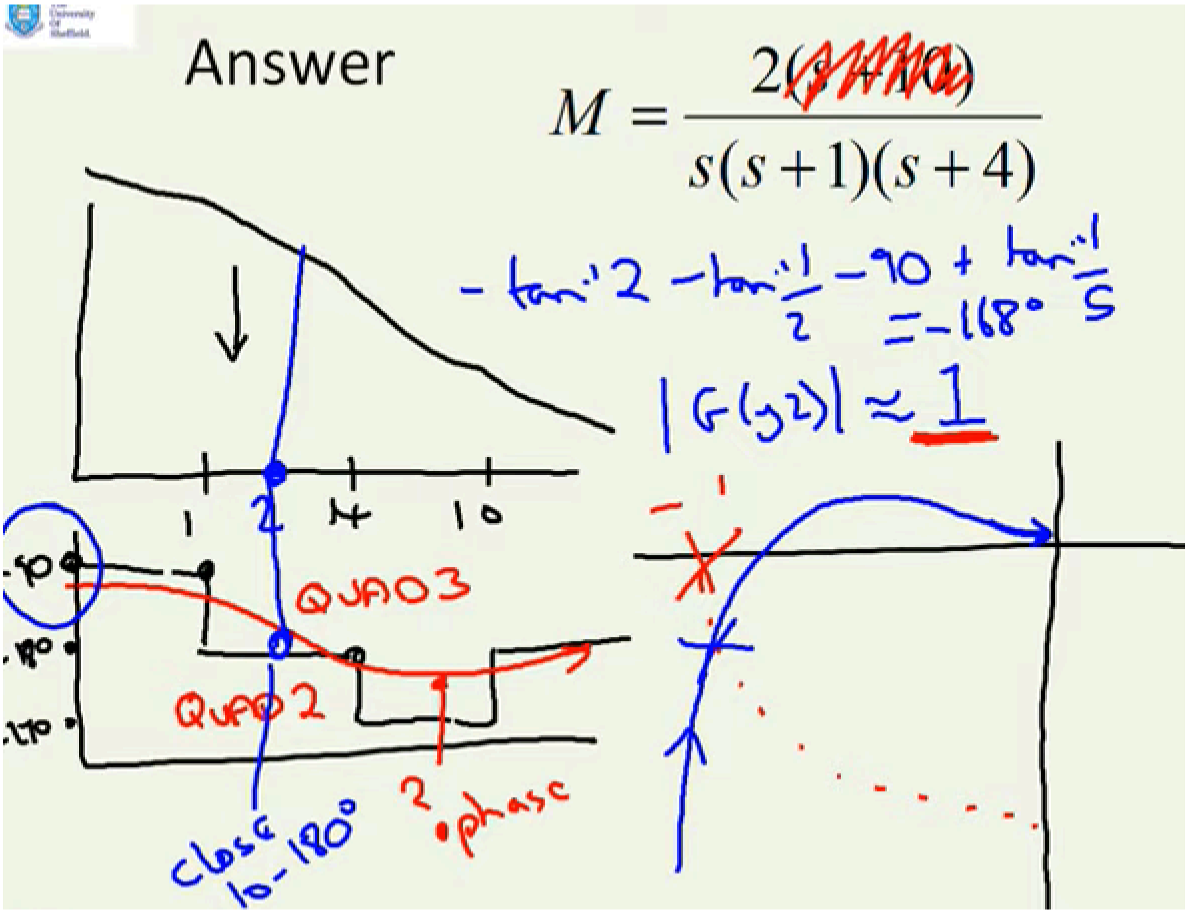
Example 14

$H = \frac{10(s+2)}{(s+5)^2}$



Example 14

$M = \frac{2(s+2)}{s(s+1)(s+4)}$



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