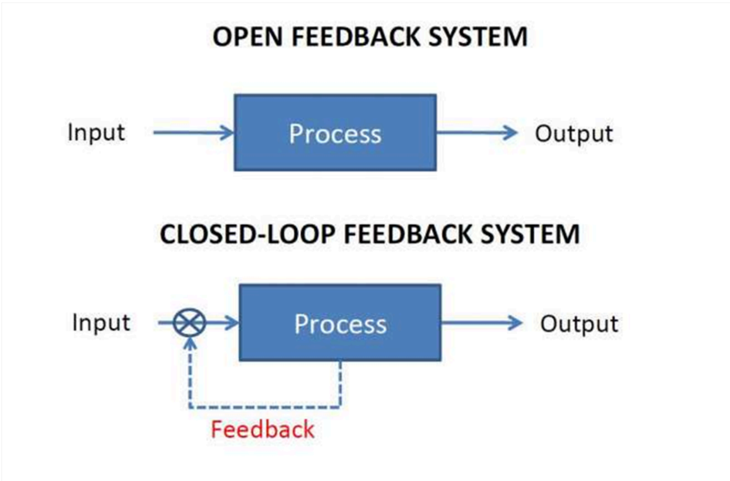
Stability of Closed Loop Systems

open-loop control in motion systems means that there is no position feedback of a moving object e.g a dishwasher or sprinkler system for your lawn

Closed-loop control means that there is some kind of position information that is fed back to the motion controller of a system and that is used in the positioning process. e.g dishwasher with a clean sensor, car with a speedometer.



Absolutely stable system.

The open loop control system is absolutely stable if all the poles of the open loop transfer function present in left half of ‘s’ plane. Similarly, the closed loop control system is absolutely stable if all the poles of the closed loop transfer function present in the left half of the ‘s’ plane.

Routh-Hurwitz criterion

All roots lie in LHP if and only if a certain set of algebraic combinations of the coefficients all have the same sign.

Example 1

a). For the transfer function $H(S) = \frac{N(S)}{D(S)}$ where must the roots of DS be located?

left-hand side of the argand diagram.

b) Transfer function $H(s) = \frac{1}{s+a}$ where a is positive. What is the $\mathscr{L}^{-1}\{H(s)\}\mathrm{L}^{-1}\{H(s)\}$?

$\mathscr{L}^{-1}\{H(s)\} = e^{-at}\mathrm{L}^{-1}\{H(s)\} = e^{-at}$

c) is the system stable?

As t goes to infinity the function goes to 0.

Example 2

Transfer function $H(s) = \frac{1}{(s+1)(s+2)(s-3)}$

a) Express as a partial fraction.

$$H(s) = \frac{A}{(s+1)} + \frac{B}{(s+3)} + \frac{C}{(s-2)}$$

c)

What is the $\mathscr{L}^{-1}\{H(s)\}$L $^{-1}\{H(s)\}$?

$$\mathscr{L}^{-1}\{H(s)\} = Ae^{-t} + Be^{-3t} + Ce^{2t} \mathrm{L}^{-1}\{H(s)\} = Ae^{-t} + Be^{-3t} + Ce^{2t}$$

c) is the system stable?

The third term makes the entire system unstable.

Example 3

Transfer function $H(s) = \frac{1}{s^4+3s^3-5s^2+s+2}$. Assess whether this system is stable.

Since not all the signs have the same sign so therefore unable.

Example 4

Used RH array to assess whether the transfer function $\frac{1}{s^4+2s^3+3s^2+10s+8}$ is stable

s^4	1	3	8
s^3	2	10	0
s^2	-2	8	
s^1	18	0	
s^0	8		

```
\(\begin{bmatrix}
s^4 & 1 & 3 & 8 \\
\hline\hline
s^3 & 2 & 10 & 0 \\
\hline
s^2 & -2 & 8 &  \\
\hline
s^1 & 18 & 0 &  \\
\hline
s^0 & 8 & & \\
\hline
\end{bmatrix}\)
```

Since there is 2 Roots in the right half plane then we can deduce that this is unstable - we only need at least 1 root in the right half plane to conclude unstable.

Example 5

Used RH array to assess whether the transfer function $\frac{1}{s^4+2s^3+3s^2+4s+5}$ is stable

$$\begin{bmatrix} s^4 & 1 & 3 & 5 \\ \hline s^3 & 2 & 4 & 0 \\ \hline s^2 & 1 & 5 & \\ \hline s^1 & -6 & 0 & \\ \hline s^0 & 5 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^4 & 1 & 3 & 5 \\
\hline\hline
s^3 & 2 & 4 & 0 \\
\hline
s^2 & 1 & 5 &  \\
\hline
s^1 & -6 & 0 &  \\
\hline
s^0 & 5 & & \\
\hline
\end{bmatrix}\)
```

Since there is 2 Roots in the right half plane then we can deduce that this is unstable

Example 6

Used RH array to assess whether the transfer function $\frac{1}{s^4+2s^3+2s^2+4s+5}$ is stable

$$\begin{bmatrix} s^4 & 1 & 2 & 5 \\ \hline s^3 & 2 & 4 & 0 \\ \hline s^2 & \epsilon & 5 & \\ \hline s^1 & \frac{4\epsilon-10}{\epsilon} & 0 & \\ \hline s^0 & 5 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^4 & 1 & 2 & 5 \\
\hline\hline
s^3 & 2 & 4 & 0 \\
\hline
s^2 & \epsilon & 5 &  \\
\hline
s^1 & \frac{4\epsilon -10}{\epsilon} & 0 &  \\
\hline
s^0 & 5 & & \\
\hline
\end{bmatrix}\)
```

Apply $\lim_{\epsilon \rightarrow 0+}$ \(\lim_{\epsilon \rightarrow 0+}\)

s^4	1	2	5
s^3	2	4	0
s^2	0+	5	
s^1	$-\infty$	0	
s^0	5		

```
\(\begin{bmatrix}
s^4 & 1 & 2 & 5 \\
\hline\hline
s^3 & 2 & 4 & 0 \\
\hline
s^2 & 0+ & 5 &  \\
\hline
s^1 & -\infty & 0 &  \\
\hline
s^0 & 5 & & \\
\hline
\end{bmatrix}\)
```

Since there is 2 Roots in the right half plane then we can deduce that this is unstable

Example 6

Used RH array to assess whether the transfer function $\frac{1}{s^4+s^3+s^2+s+1}$ is stable

s^4	1	1	1
s^3	1	1	0
s^2	ϵ	1	
s^1	$\frac{\epsilon-1}{\epsilon}$	0	
s^0	1		

```
\(\begin{bmatrix}
s^4 & 1 & 1 & 1 \\
\hline\hline
s^3 & 1 & 1 & 0 \\
\hline
s^2 & \epsilon & 1 &  \\
\hline
s^1 & \frac{\epsilon -1}{\epsilon} & 0 &  \\
\hline
s^0 & 1 & & \\
\hline
\end{bmatrix}\)
```

Apply $\lim_{\epsilon \rightarrow 0+}$ \(\lim_{\epsilon \rightarrow 0+}\)

s^4	1	1	5
s^3	1	1	0
s^2	0+	1	
s^1	$-\infty$	0	
s^0	1		

```
\(\begin{bmatrix}
s^4 & 1 & 1 & 5 \\
\hline\hline
s^3 & 1 & 1 & 0 \\
\hline
s^2 & 0+ & 1 &  \\
\hline
s^1 & -\infty & 0 &  \\
\hline
s^0 & 1 & & \\
\hline
\end{bmatrix}\)
```

```
Since there is 2 Roots in the right half plane then we can deduce that this is unstable
```

Example 7

Used RH array to assess whether the transfer function $\frac{1}{s^4+3s^3+3s^2+2s+1}$ is stable

$$\begin{bmatrix} s^4 & 1 & 3 & 5 \\ \hline s^3 & 3 & 2 & 0 \\ \hline s^2 & \frac{7}{3} & 1 & \\ \hline s^1 & \frac{5}{7} & & \\ \hline s^0 & 1 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^4 & 1 & 3 & 5 \\
\hline\hline
s^3 & 3 & 2 & 0 \\
\hline
s^2 & \frac{7}{3}& 1 &  \\
\hline
s^1 & \frac{5}{7}&  &  \\
\hline
s^0 & 1 & & \\
\hline
\end{bmatrix}\)
```

```
All the elements of the first column of the Routh array are positive. There is no sign change
in the first column of the Routh array. So, the control system is stable.
```

Example 8

Used RH array to assess whether the transfer function $\frac{1}{s^4+2s^3+s^2+2s+1}$ is stable

$$\begin{bmatrix} s^4 & 1 & 1 & 1 \\ \hline s^3 & 2 & 2 & 0 \\ \hline s^2 & \epsilon & 1 & \\ \hline s^1 & \frac{\epsilon-1}{\epsilon} & & \\ \hline s^0 & 1 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^4 & 1 & 1 & 1 \\
\hline\hline
s^3 & 2 & 2 & 0 \\
\hline
s^2 & \epsilon & 1 &  \\
\hline
s^1 & \frac{\epsilon -1}{\epsilon} &  &  \\
\hline
s^0 & 1 & & \\
\hline
\end{bmatrix}\)
```

Apply $\lim_{\epsilon \rightarrow 0+}$\(\lim_{\epsilon \rightarrow 0+}\)

$$\begin{bmatrix} s^4 & 1 & 1 & 5 \\ \hline s^3 & 2 & 2 & 0 \\ \hline s^2 & 0+ & 1 & \\ \hline s^1 & -\infty & 0 & \\ \hline s^0 & 1 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^4 & 1 & 1 & 5 \\
\hline\hline
s^3 & 2 & 2 & 0 \\
\hline
s^2 & 0+ & 1 &  \\
\hline
s^1 & -\infty & 0 &  \\
\hline
s^0 & 1 & & \\
\hline
\end{bmatrix}\)
```

Since there is 2 Roots in the right half plane then we can deduce that this is unstable

Example 9

Used RH array to assess whether the transfer function $\frac{1}{s^3-3s+2}$\(\frac{1}{s^3-3s+2}\) is stable

$$\begin{bmatrix} s^3 & 1 & -3 \\ \hline s^2 & \epsilon & 2 \\ \hline s^1 & \frac{-3\epsilon-2}{\epsilon} & 0 \\ \hline s^0 & 2 & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^3 & 1 & -3  \\
\hline
s^2 & \epsilon & 2 &  \\
\hline
s^1 & \frac{-3\epsilon-2}{\epsilon} & 0 &  \\
\hline
s^0 & 2 & & \\
\hline
\end{bmatrix}\)
```

As $\epsilon \rightarrow 0$\(\epsilon \rightarrow 0\) $\frac{-3\epsilon-2}{\epsilon}$\(\frac{-3\epsilon - 2}{\epsilon}\) becomes negativ
e so there are two sign
changes.

Example 10

Used RH array to assess whether the transfer function $\frac{1}{s^5+3s^4+s^3+3s^2+s+3}$
\(\frac{1}{s^5+3s^4+s^3+3s^2+s+3}\) is stable

$$\begin{bmatrix} s^5 & 1 & 1 & 1 \\ \hline s^4 & 3 & 3 & 3 \\ \hline s^3 & 0 & 0 & \\ \hline s^2 & 2 & & \end{bmatrix}$$

```
\(\begin{bmatrix}
s^5 & 1 & 1 & 1\\
\hline
s^4 & 3 & 3 & 3 \\
\hline
s^3 & 0 & 0 &  \\
\hline
s^2 & 2 & & \\
\hline
\end{bmatrix}\)
```

$A(s) = 3s^4 + 3s^2 + 3$\(A(s) = 3s^4 + 3s^2 + 3 \)
$\frac{dA}{dS} = 12s^3+6s$\(\frac{dA}{dS} = 12s^3+6s \)

```
\(\begin{bmatrix}
s^5 & 1 & 1 & 1\\
\hline
s^4 & 3 & 3 & 3 \\
\hline
s^3 & 12 & 6 &  \\
\hline
s^2 & \frac{36-18}{12} & 3 & \\
\hline
s & \frac{9-36}{1.5} &  & \\
\hline
s^{0} & 3 &  & \\
\end{bmatrix}\)
```

$$\begin{bmatrix} s^5 & 1 & 1 & 1\\ \hline s^4 & 3 & 3 & 3 \\ \hline s^3 & 12 & 6 & \\ \hline s^2 & \frac{36-18}{12} & 3 & \\ \hline s & \frac{9-36}{1.5} & & \\ \hline s^{0} & 3 & & \end{bmatrix}$$

```
There are two sign changes in the first column of Routh table. Hence, the control system is u
nstable.
```

$$\left(s^5+3s^4+s^3+3s^2+s+3\right)=\frac{1}{3}\left(3s^4+3s^2+3\right)(s+3)$$

$$\left(s^5+3s^4+s^3+3s^2+s+3\right)=\frac{1}{3}\left(3s^4+3s^2+3\right)(s+3)$$

Example 10

Use RH array to assess whether the transfer function $\frac{1}{s^5+2s^4+24s^3+48s^2-25s-50}$ $\frac{1}{s^5+2s^4+24s^3+48s^2-25s-50}$ is stable

$$\begin{bmatrix} s^5 & 1 & 24 & -25 \\ \hline s^4 & 2 & 48 & -50 \\ \hline s^3 & 0 & 0 & \end{bmatrix}$$

$$\begin{bmatrix} s^5 & 1 & 24 & -25 \\ s^4 & 2 & 48 & -50 \\ s^3 & 0 & 0 & \end{bmatrix}$$

$A(s)=2s^4+48s^2-50$$A(s)=2s^4+48s^2-50$

$\frac{dA}{dS}=8s^3+96s$$\frac{dA}{dS}=8s^3+96s$

$$\begin{bmatrix} s^5 & 1 & 24 & -25 \\ \hline s^4 & 2 & 48 & -50 \\ \hline s^3 & 8 & 96 & \\ \hline s^2 & 24 & -50 & \\ \hline s & 112.7 & 0 & \\ s^0 & -50 & 0 & \end{bmatrix}$$

$$\begin{bmatrix} s^5 & 1 & 24 & -25 \\ s^4 & 2 & 48 & -50 \\ s^3 & 8 & 96 & \\ s^2 & 24 & -50 & \\ s & 112.7 & 0 & \\ s^0 & -50 & 0 & \end{bmatrix}$$

We see that there is one change in the sign in the first column

$$s^5+2s^4+24s^3+48s^2-25s-50=\frac{1}{2}\left(2s^4+48s^2-50\right)(s+2)$$

$$s^5+2s^4+24s^3+48s^2-25s-50=\frac{1}{2}\left(2s^4+48s^2-50\right)(s+2)$$

Example 11

Use RH array to assess whether the transfer function $\frac{1}{s^5+s^4+2s^3+2s^2+1s+1}$ $\frac{1}{s^5+s^4+2s^3+2s^2+1s+1}$ is stable

$$\left[\begin{array}{c|ccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{cccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 0 & 0 & \end{array}\right]$$

$A(s)=s^4+2s^2+1$ $A(s)=s^4+2s^2+1$

$\frac{dA}{dS}=4s^3+4s$ $\frac{dA}{dS}=4s^3+4s$

$$\left[\begin{array}{c|ccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 4 & 4 & \\ s^2 & 1 & 1 & \\ s & 0 & 0 & \end{array}\right]$$

$$\left[\begin{array}{cccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 4 & 4 & \\ s^2 & 1 & 1 & \\ s & 0 & 0 & \end{array}\right]$$

$A(s)=s^2+1$ $A(s)=s^2+1$

$\frac{dA}{dS}=2s$ $\frac{dA}{dS}=2s$

$$\left[\begin{array}{c|ccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 4 & 4 & \\ s^2 & 1 & 1 & \\ s & 2 & 0 & \\ s^0 & 1 & 0 & \end{array}\right]$$

$$\left[\begin{array}{cccc} s^5 & 1 & 2 & 1 \\ s^4 & 1 & 2 & 1 \\ s^3 & 4 & 4 & \\ s^2 & 1 & 1 & \\ s & 2 & 0 & \\ s^0 & 1 & 0 & \end{array}\right]$$

We see that there is no change in the sign in the first column

$$\left(s^5+s^4+2s^3+2s^2+1s+1\right)=\left(s^2+2\right)^2(s+1)$$

$$\left(s^5+s^4+2s^3+2s^2+1s+1\right)=\left(s^2+2\right)^2(s+1)$$

Example 12

Used RH array to assess whether the transfer function $\frac{1}{s^3+3Ks^2+(K+2)s+4}$ is stable

$$\left[\begin{array}{cccc} s^3 & 1 & (K+2) & 0 \\ \hline s^2 & 3K & 4 & \\ \hline s^1 & \frac{(K+2)(3K)-4}{3K} & & \\ \hline s^0 & 4 & & \\ \hline \end{array}\right]$$

```
\(\begin{bmatrix}
s^3 & 1 & (K+2) & 0 \\
\hline
s^2 & 3K & 4 &  \\
\hline
s^1 & \frac{(K+2)(3K) -4}{3K} &  &  \\
\hline
s^0 & 4  & & \\
\hline
\end{bmatrix}\)
```

$3K > 0$ `\( 3K \gt 0\)`

$\frac{(K+2)(3K)-4}{3K} > 0$ `\(\frac{(K+2)(3K) -4}{3K}\gt 0\)`

$3K^2+6K-4 \geq 0$ $3K^2+6K-4 \geq 0$

$k > \frac{-6+\sqrt{84}}{6}$ $k > \frac{-6+\sqrt{84}}{6}$

$k \geq -1+\frac{\sqrt{21}}{3}$

Example 13

Is $\frac{1}{s^2+s+1}$ stable

$s = \frac{-1 \pm i\sqrt{3}}{2}$ $s = \frac{-1 \pm i\sqrt{3}}{2}$

$$\left[\begin{array}{ccc} s^2 & 1 & 1 \\ \hline s^1 & 1 & 0 \\ \hline s^0 & 0.5 & \end{array}\right]$$

$$\begin{bmatrix} s^2 & 1 & 1 \\ s^1 & 1 & 0 \\ s^0 & 0.5 & \end{bmatrix}$$

No sign change so stable.

Example 14

Is $\frac{1}{s^4+s^2+1}$ $\frac{1}{s^4+s^2+1}$ stable

$t^2 = \frac{-1 \pm i\sqrt{3}}{2}$ $t^2 = \frac{-1 \pm i\sqrt{3}}{2}$

$t^2 = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$$t^2 = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$

$r^2[cos(\alpha) + isin(\alpha)]^2 = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$$r^2[cos(\alpha) + isin(\alpha)]^2 = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$

$r^2[cos(2\alpha) + isin(2\alpha)] = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$$r^2[cos(2\alpha) + isin(2\alpha)] = 1[cos(\frac{2\pi}{3}) + isin(\frac{2\pi}{3})]$

$\alpha_1 = \frac{\pi}{3}$$\alpha_1 = \frac{\pi}{3}$

$\alpha_2 = -\frac{\pi}{3}$$\alpha_2 = -\frac{\pi}{3}$

$\alpha_3 = -\frac{2\pi}{3}$$\alpha_3 = -\frac{2\pi}{3}$

$\alpha_4 = \frac{2\pi}{3}$$\alpha_4 = \frac{2\pi}{3}$

https://www.youtube.com/watch? (https://www.youtube.com/watch?)

v=yf09OrHa520&list=PLUMWjy5jgHK3j74Z5Tq6Tso1fSfVWZC8L&index=9

https://www.ssgmce.ac.in/student_resource/Electronics%20&%20Telecommunication%20Engg./CSE_3u/routh%20criterion%202.pdf (https://www.ssgmce.ac.in/student_resource/Electronics%20&%20Telecommunication%20Engg./CSE_3u/routh%20criterion%202.pdf)

https://uomustansiriyah.edu.iq/media/lectures/5/5_2020_05_06!01_12_34_PM.pdf