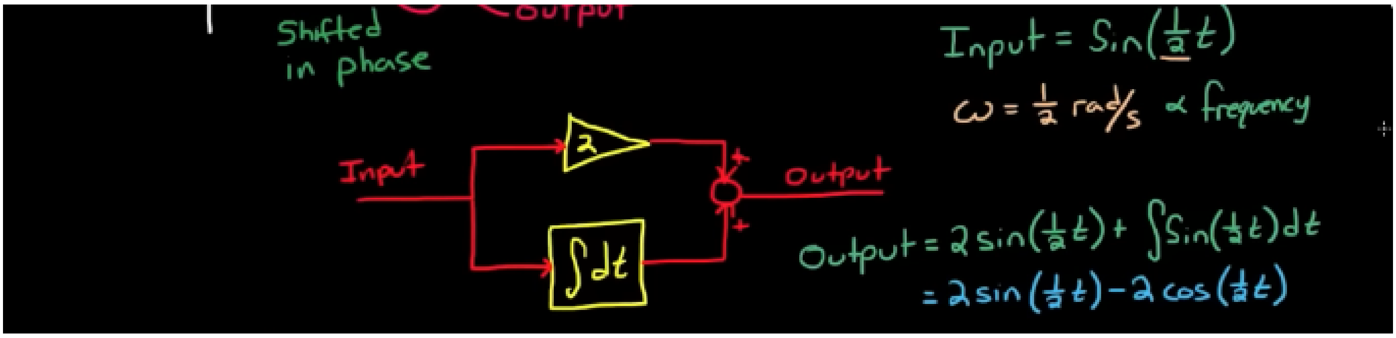
Bode Plot

Poles is the values of s that causes \(H(s) \rightarrow \infty \)

Zeroes is the values of s that causes \(H(s) \rightarrow 0 \)

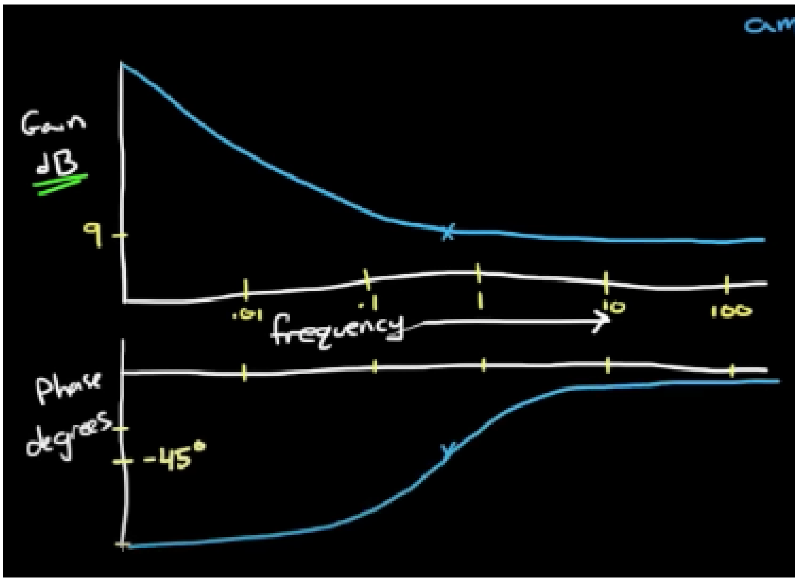
A sine wave input will always generate a sine wave out - just amplitude and phase shift will change.

Example 1

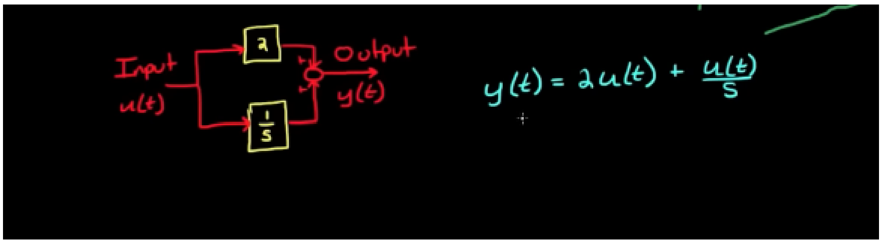


Output \(\approx 2.83 sin(\frac{1}{2}t-0.785)\)

$$dB = 20log_{10}\frac{A}{A_0} = 20\log_{10}\frac{2.83}{2}$$



Transfer function



$y(t) =\frac{2s+1}{s} u(t)$

$y(t) =\frac{2\omega j+1}{\omega j} u(t)$

$y(t) =[2 - \frac{1}{\omega} j] u(t)$

Amplitude $= \sqrt{2^2+\frac{1}{\omega^2}}$

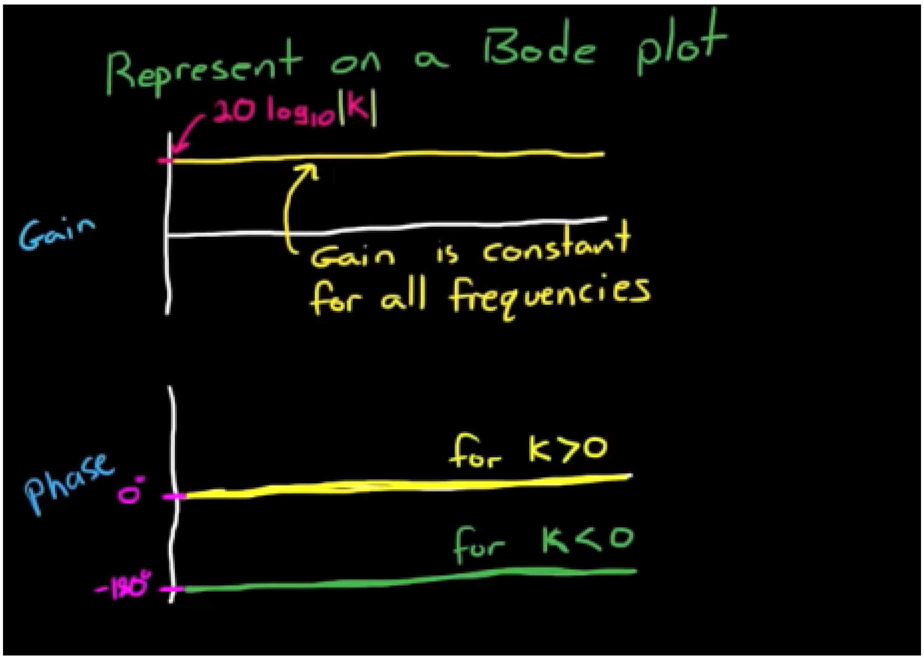
Example 2

$H(s) =k$

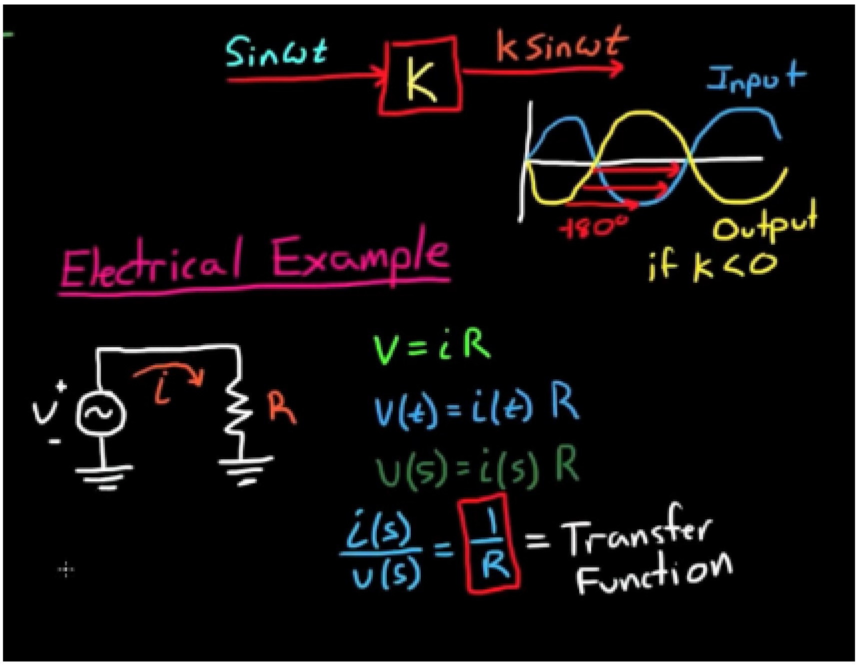
gain $=|H(s)| = k$

phase = 0 when $k \gt 0$

phase = 180 when $k \lt 0$



Example of a constant transfer function



Example 3

Transfer function $\frac{1}{s}$

$H(j\omega) = \frac{1}{j\omega} = -\frac{j}{\omega}$

gain $= \frac{1}{\omega}$

phase $= \arg (- \frac{1}{\omega} ,0) = -90$ degrees

Example 5

Transfer function $5\frac{1}{s}$

Magnitude $= 20\log_{10} [\frac{5}{w}]$

Magnitude $= 20\log_{10} [5]+ 20\log \frac{1}{w}$

Example 6

Transfer function $3\frac{1}{s^2}$

Magnitude $= 20\log_{10} [\frac{3}{w^2}]$

Magnitude $= 20\log_{10} [3]+ 20\log \frac{1}{w}+ 20\log \frac{1}{w}$\)

Example 8

Zeros of $s = \frac{1}{\frac{1}{s}}$

$-20log (H(j\omega)) = -20log (|\frac{1}{\frac{1}{j\omega}}|$

$-20log (H(j\omega)) = -20log (|\frac{j}{\frac{1}{\omega}}|$

$-20log (H(j\omega)) = 20log (\frac{1}{\omega})$

Phase = arg $(j\omega) = 90$ degrees

Example 7

Transfer function $\frac{1}{1+ \frac{s}{w_0}}$

Transfer function $\frac{1}{1+ \frac{j\omega}{w_0}}$

Transfer function = $\frac{1}{1+\frac{\omega^2}{w_0^2}} - j\frac{\frac{\omega}{\omega_0}}{1+\frac{\omega^2}{w_0^2}}$

Magnitude $= 20\log_{10} [H(\omega j)] = -20\log_{10} \sqrt{1+\frac{\omega^2}{\omega_{0}^2}}$

Phase $= \arctan \frac{-\omega}{\omega_0}$

Example 8

$H(s) = \frac{\omega_{0}^2}{s^2+2\gamma \omega_{0} s+\omega_{0}^2}$

\(H(s) = \frac{1}{\frac{s}{\omega_{0}}^2+2\gamma \frac{s}{\omega_{0}} +1} \)

\(H(j\omega) = \frac{1}{\frac{j\omega}{\omega_{0}}^2+2\gamma \frac{j\omega}{\omega_{0}} +1} \)

$$H(j\omega) = \frac{1-[\frac{\omega}{\omega_0}]^2]}{[1-[\frac{\omega}{\omega_0}]^2]^2+[2\gamma\frac{\omega}{\omega_0}]^2} - j\frac{2\gamma\frac{\omega}{\omega_0}}{[1-[\frac{\omega}{\omega_0}]^2]^2+[2\gamma\frac{\omega}{\omega_0}]^2}$$

Case 1. \(\omega \leq \omega_{0} \)

\(\frac{\omega}{\omega_{0}}\) is very small

\(|H(j\omega)| = \sqrt{1^2+0^2} = 1\)

In decimals \(= 20\log_{10} 1 = 0\)

Phase =

Case 2. \(\omega \geq \omega_{0} \)

is very big

In decimals

Phase =

Phase = = -180 degrees

Case 3 \(\omega = \omega_{0} \)

In decibels \(= 20log_{10} (2\gamma)^{-1}\)

In decibels \(= -20log_{10} (2\gamma)\)

Phase \(= arg(H(jw)) = arctan(\frac{0}{-(2\gamma)^{-1}})= -90\)